

Temperature determination from the lattice gas model

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Abstract

Determination of temperature from experimental data has become important in searches for critical phenomena in heavy ion collisions. Widely used methods are ratios of isotopes (which rely on chemical and thermal equilibrium), population ratios of excited states etc. Using the lattice gas model we propose a new observable: n_{ch}/Z where n_{ch} is the charge multiplicity and Z is the charge of the fragmenting system. We show that the reduced multiplicity is a good measure of the average temperature of the fragmenting system.

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The extraction of a temperature from observables in a fragmenting system is a much studied problem in heavy ion collisions. It was originally estimated from the slopes of inclusive spectra of charged particles. However, such calibrations are suspect because of collective flow and other dynamical effects. Other procedures were proposed. One method is to extract temperatures from populations of excited states of emitted fragments[1]. The practical problem here is that of detector efficiency. If chemical and thermal equilibria are achieved, one may obtain temperature information more easily from a double isotope ratio defined by[2]

$$R = \frac{Y(A_i, Z_i)/Y(A_i + \Delta A, Z_i + \Delta Z)}{Y(A_j, Z_j)/Y(A_j + \Delta A, Z_j + \Delta Z)} \quad (1)$$

where $Y(A_i, Z_i)$ is the total yield of the emitted fragment with mass and charge number A_i, Z_i . One chooses ΔA and ΔZ to be the same for both the numerator and denominator to cancel out the effects of proton and neutron chemical potentials. This method was used in a recent paper[3]. In that work it was found that as a function of E^*/A , the excitation energy per particle, the temperature was particularly flat around $T=5$ MeV. This implied a maximum in the specific heat which could be the signature of a phase transition. This observation has sparked renewed interest in the methods that were used to extract temperatures [4-7]

Careful investigations reveal that the deduced temperature may depend upon the particular nuclei used in eq. (1). Thus ambiguities remain. These ambiguities arise from effects of sequential decay and other processes neglected by the simple theoretical model of Ref. [2]. It is extremely difficult to calculate such corrections from theory hence a phenomenological approach was taken in Ref. [4] to estimate corrections. That analysis reduces much of the ambiguity.

In this communication we propose the reduced multiplicity $m = n_{ch}/Z$ as a thermometer, where n_{ch} is the charge particle multiplicity and Z is the charge of the fragmenting system. At zero temperature m is of order zero and at high temperature it approaches 1. If m monotonically increases with T , one can attempt to extract T from a measurement of m .

There are several reasons why this needs be studied.

(1) The temperature extracted in this approach is in some sense an averaged temperature exploiting the properties of all fragments rather than a selected few which are used as thermometers.

(2) Calculations show that m is at least nearly, if not totally, independent of the size of the fragmenting system. Thus we can use it for a range of systems of different sizes.

(3) In other investigations a parametrization of m in terms of T may become useful for estimating critical exponents[5].

(4) Success of the lattice gas model depends on the ability of predicting correctly overall yields at freeze-out density as the excitation energy per particle (which is related to temperature) changes. Our past successes in correlating with data[8-10] suggest that the proposed method will be a reasonable measure of temperature.

(5) An important question is whether the numbers of clusters which are formed in such models will change a great deal by sequential decay. We like to point out that the formation of clusters in the lattice gas model is quite different from that in some other models, for example, the Copenhagen model[11]. In the latter clusters are formed at a given temperature. A cluster at a finite temperature will have components of many excited states and many of these excited states can emit particles. In the lattice gas model this is done differently. From a global temperature we first generate by Monte-Carlo simulation momentum of each nucleon. We then decide with these momenta whether two nucleons will bind or not. Thus our clusters do not have a temperature; they instead have a well-defined excitation energy. Within rules of classical mechanics they will not decay. The formula we use to deduce cluster sizes reduces a truly many-body problem into independent sums of two body problems. This is an approximation. There will be scenarios where our prescription for clusters will overestimate the size of a cluster. But there are also scenarios where our formula, for example, obtains two clusters, whereas the two clusters will actually coalesce into one. Lattice gas calculation for clusters was compared with a many body molecular dynamics calculation in Ref. [12] and statistically negligible difference was found.

One might argue that actual nuclei are quantum mechanical objects and classical calculations should be followed by a ad-hoc correction. In this work however we stay entirely within the classical rules followed in the model and obtain the consequences.

The lattice gas model has been described before[8,9]. In our previous calculations we had used the same value of interaction ϵ between neutron-proton, neutron-neutron and proton-proton. An attractive interaction was used. We now use an attractive interaction ϵ_{np} between neutron-proton but the bond between neutron-neutron and proton-proton should be either zero or slightly repulsive. This eliminates unphysical clusters like di-neutrons or di-protons and makes the model much more realistic. In accordance with a force much used in molecular dynamics calculation for nuclear collisions[13] we have used a slightly repulsive potential for neutron-neutron and proton-proton bond. Thus $\epsilon_{np} = \epsilon$ and $\epsilon_{nn} = \epsilon_{pp} = -\epsilon/5$. The value of ϵ is negative: ϵ is the only energy scale in the lattice gas model. The value of ϵ can be chosen from binding energy considerations. In an infinite nuclear matter ϵ is about -5.33 MeV. In real finite nuclei because of Coulomb interaction and surface effects a lower value is expected to get the correct binding energy. As in the case of one type of bonds the thermal critical temperature appears at $T_c = -1.1275\epsilon$ [12]. We can thus either use ϵ or T_c as an energy unit. We will use the latter.

Fig. 1 shows the dependence of temperature as a function of the reduced multiplicity $m = n_{ch}/Z$ for two systems with different mass number, A , and charge number Z : $A=137$, $Z=57$ and $A=87$, $Z=37$. In the calculation we assumed a freeze-out density $\rho_f \approx 0.4\rho_0$ where ρ_0 is the density of normal nuclear matter. As shown by the solid and dashed lines in the figure, the temperature is largely independent of the size of the fragmenting system.

To explore the effect of freeze-out density on the temperature dependence on m , two Monte Carlo calculations for the fragmenting system with $A=137$, $Z=57$ for $0.26\rho_0$ (solid lines) and $0.4\rho_0$ (dashed lines) are plotted in Figure 2. The two densities correspond to the freeze-out density of light fragmenting systems[10] and medium fragmenting systems[8,9] Notice that dm/dT goes to zero at both $T = 0$ and $T = \infty$ limit. All densities give the same high temperature limit. The reason that the slope is zero at the low temperature limit is

that a certain amount of energy will have to be supplied to the fragmenting system before it will split into two or more parts. This is a quantum effect of the lattice gas model due to the quantization of binding energy and lattice space.

To compare with experimental data, it would be desirable to have a simple formula so that for a given value of m one can readily deduce the temperature given by this model. Motivated by this we have tried a simple parametrization;

$$T/T_c = a \ln \frac{b\sqrt{m}}{1 - \sqrt{m}} \quad (2)$$

The results of such parametrization $a = 1/7.5$, $b = 110$; and $a = 1/5.3$, $b = 40$ for low and high density respectively, are shown as the two dot-dashed lines in Fig. 2. The parametrization agrees with the Monte-Carlo simulations up to $m = 0.8$. This is well within the range of temperature that are of current interest. We will ignore the region beyond $m = 0.8$ here. Similarly we ignore the region of very low m ($\sqrt{m} < 1/b$).

Recently, temperatures have been determined as a function of total charge particle multiplicities (n_{tot}) for the Au+C system at 1 GeV/A incident energies[5]. The analysis also extracted the fragment source size (A) and the multiplicities (n_{ch}) attributed to the emitted fragment source. Assuming the fragment source has the same neutron to proton ratio as the Au projectile, the following relations are obtained from Ref. [5],

$$n_{ch} = 0.3n_{tot} \quad (3)$$

$$Z = -0.68n_{tot} + 80.5 \quad (4)$$

As the temperatures measured with the hydrogen and helium isotope ratios were more reliable and suffered less systematic uncertainties and sequential decay correction effects, we have plotted these data in Fig. 2 by assuming $T_c = 8.3$ MeV. The critical temperature T_c is estimated to be about 4-6 MeV depending on the size of the lattice system. In the present study, for simplicity, T_c is treated as an adjustable parameter. The slope of the data agrees with the dashed line which is the prediction for breakup density of $0.4\rho_0$. Over the region

of $n_{ch}/Z=0.1$ to 0.4, there is good agreement between data and calculations. At very low multiplicities, the data deviates from the calculations. Instead of dropping rapidly to zero as in the calculations, the slope of the data remains nonzero. This could arise from the inability to accurately deduce both the multiplicity and charge of the fragment source for very peripheral collisions.

In summary, we have shown that the reduced charged particle multiplicity can be used to extract the average temperature of the fragmenting system. The extracted empirical parametrizations for eq. (2) that relates the temperature to the reduced charged particle multiplicity for the Au+C system at 1 GeV/A is $a = 1/5.3$, $b = 40$ and $T_c = 8.3$ MeV.

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REFERENCES

- [1] C. Schwarz et. al. Phys. Rev. C48, 676 (1993).
- [2] S. Albergo et. al., Nuove Cimento, 89, 1 (1985).
- [3] J. Pochodzalla et. al, Phys. Rev. Lett. 75, 1040 (1995).
- [4] M. B. Tsang, W. G. Lynch, H. Xi and W. A. Friedman, MSUCL-1035.
- [5] J. A. Hauger, Phys. Rev. Lett., 77, 235 (1996).
- [6] M. B. Tsang et. al, Phys. Rev. C53, R1057 (1996).
- [7] A. Kolomeits et al., Phys. Rev. C (in press).
- [8] J. Pan and S. Das Gupta, Phys. Lett. B344, 29 (1995).
- [9] J. Pan and S. Das Gupta, Phys. Rev. C51, 1384 (1995).
- [10] L. Beaulieu et. al., Phys. Rev. C (in press).
- [11] J. P. Bondorf, A. S. Botvina, A. S. Ijilinov, I. N. Mishustin and K. Sneppen, Phys. Rept. 257, 133 (1995).
- [12] S. Das Gupta and J. Pan, phys. Rev. C53, 1319 (1996).
- [13] R. J. Lenk, T. J. Schlagel and V. R. Pandharipande, Phys. Rev. C52, 271 (1990).
- [14] R. B. Stinchcombe in: *Phase transitions and critical phenomena, vol 7*, eds C. Domb and J. L. Lebowitz (Academic Press, London, 1983).

FIGURE CAPTIONS

Fig. 1 Dependence of T/T_c as a function of n_{ch}/Z for fixed freeze out density $\rho_f \approx 0.4\rho_0$ for two different fragmenting systems.

Fig. 2 T/T_c as a function of n_{ch}/Z for two different freeze-out densities. The solid and dashed lines are Monte Carlo simulations with the Lattice gas models. The two dot dashed lines are fits with eq. (2). Data points are the multiplicity data of Au+C reaction [5]. See text for detail.

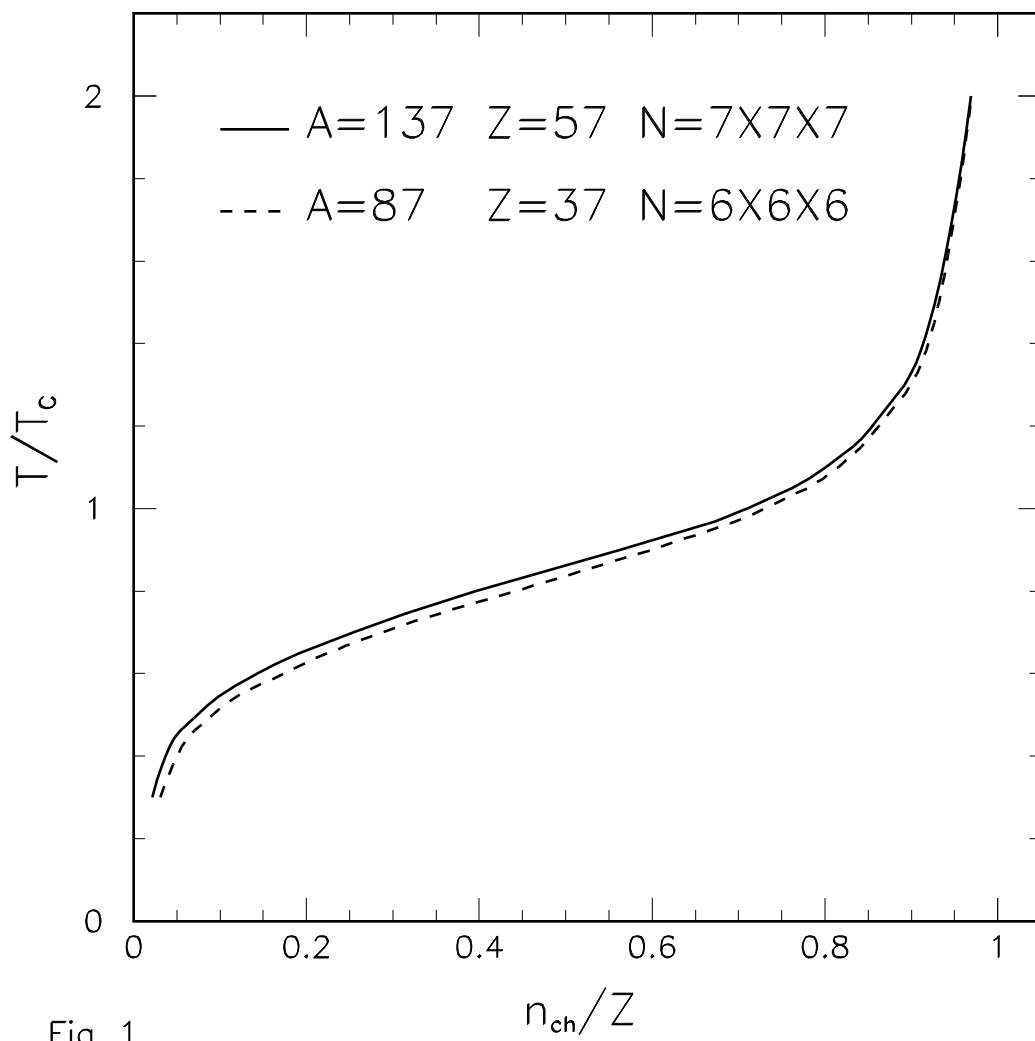


Fig. 1

